BOUNDARY LAYER OF TWO-TEMPERATURE PLASMA ON ELECTRODES OF MHD CHANNEL WITH CROSSED ELECTRIC AND MAGNETIC FIELDS FOR LARGE VALUES OF THE HALL PARAMETER

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The rational organization of the cycle in MHD devices requires, first of all, knowledge of the laws of interaction of the plasma stream with the electrodes. Therefore the problem of the plasma boundary layer on the electrodes of the MHD channel is of considerable practical interest. As a rule, there is considerable rarefaction in accelerator channels and the magnetic field intensity is rather high. This leads to the need for accounting for the Hall current fields and the influence of the Hall parameter on the plasma transport properties. Moreover, discontinuity of the electron temperature usually occurs in plasma accelerators.

Existing studies of the MHD boundary layer have concerned individual particular questions or the solution of simplified problems [1-3]. The literature does not present the complete system of equations for the boundary layer on the electrodes of a compressible magnetized two-temperature plasma.

In the following we present the boundary layer equations for the indicated general case. We consider only a completely ionized quasi-neutral plasma for small values of the magnetic Reynolds number.

We direct the x axis along the conducting wall, the y axis along the normal to the wall, and the z axis perpendicular to the x and y axes. Let the external magnetic field be characterized by the components $B_x = B_y = 0$, $B_z = B$ and the electric field which develops in the plasma by the components E_x, E_y.

The system of MHD equations for a two-dimensional plasma stream will be [4]

$$mn\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+j_yB-\frac{\partial \pi_{xx}^i}{\partial x}-\frac{\partial \pi_{xy}^i}{\partial y} \tag{1}$$

$$mn\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}-j_xB-\frac{\partial \pi_{yx}^i}{\partial x}-\frac{\partial \pi_{yy}^i}{\partial y} \tag{2}$$

$$\frac{3}{2}kn\left(u\frac{\partial T_i}{\partial x}+v\frac{\partial T_i}{\partial y}\right)=-\left(\frac{\partial q_x^i}{\partial x}+\frac{\partial q_y^i}{\partial y}\right)-p_i\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)$$
$$-\pi_{xx}^i\frac{\partial u}{\partial x}-\pi_{xy}^i\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\pi_{yy}^i\frac{\partial v}{\partial y}+3k\frac{m_e}{m}\frac{n}{\tau_e}(T_e-T_i) \tag{3}$$

$$\frac{3}{2}kn\left(u\frac{\partial T_e}{\partial x}+v\frac{\partial T_e}{\partial y}\right)=-\left(\frac{\partial q_x^e}{\partial x}+\frac{\partial q_y^e}{\partial y}\right)-p_e\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \tag{4}$$
$$-\pi_{xx}^e\frac{\partial u}{\partial x}-\pi_{xy}^e\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)-\pi_{yy}^e\frac{\partial v}{\partial y}+j_x(E_x+vB)+j_y(E_y-uB)$$
$$-\frac{3km_e}{m}\frac{n}{\tau_e}(T_e-T_i),\qquad \frac{\partial}{\partial x}(nu)+\frac{\partial}{\partial y}(nv)=0 \tag{5}$$

Here u and v are the projections of the mass-average plasma velocity on the coordinate axes; n is the number density of electrons or ions; p is the total pressure; p_e and p_i are the partial pressures; T_e and T_i are the electron and ion temperatures; m and m_e are the ion and electron masses, respectively.

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The current density projections are given by the generalized Ohm's law

$$j_x = \sigma\left\{E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x} - \frac{A_2}{A_1}\left[E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right] + \frac{k}{e}\left(\frac{A_3}{A_1}\frac{\partial T_e}{\partial x} - \frac{A_4}{A_1}\frac{\partial T_e}{\partial y}\right)\right\} \tag{6}$$

$$j_y = \sigma\left\{E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y} + \frac{A_2}{A_1}\left(E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x}\right) + \frac{k}{e}\left(\frac{A_3}{A_1}\frac{\partial T_e}{\partial y} + \frac{A_4}{A_1}\frac{\partial T_e}{\partial x}\right)\right\} \tag{7}$$

Here σ is the plasma conductivity across the magnetic field and A_k are functions of the Hall parameter H_e for the electrons. The elements of the viscous stress tensors of the ions and electrons have the form

$$\begin{aligned}
\pi_{xx}^{i} &= -\eta_i\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} + H_i\frac{5}{3}\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)\right] \\
\pi_{xy}^{i} &= -\eta_i\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} - H_i\frac{5}{3}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right] \\
\pi_{yy}^{i} &= -\eta_i\left[\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} - H_i\frac{5}{3}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \\
\pi_{xx}^{e} &= -\eta_e\left[\frac{4}{3}\frac{\partial u}{\partial x} - \frac{2}{3}\frac{\partial v}{\partial y} - H_e\frac{50}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right] \\
\pi_{xy}^{e} &= -\eta_e\left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + H_e\frac{20}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}\right)\right] \\
\pi_{yy}^{e} &= -\eta_e\left[\frac{4}{3}\frac{\partial v}{\partial y} - \frac{2}{3}\frac{\partial u}{\partial x} + H_e\frac{50}{3(2+\sqrt{2})}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)\right]
\end{aligned} \tag{8}$$

The projections of the electron and ion thermal flux vectors are

$$\begin{aligned}
q_x^{\,i} &= -\lambda_i\left(\frac{\partial T_i}{\partial x} + \frac{5}{4}H_i\frac{\partial T_i}{\partial y}\right), \quad q_y^{\,i} = -\lambda_i\left(\frac{\partial T_i}{\partial y} - \frac{5}{4}H_i\frac{\partial T_i}{\partial x}\right) \\
q_x^{\,e} &= -\lambda_e\left(\frac{\partial T_e}{\partial x} - \frac{A_6}{A_5}\frac{\partial T_e}{\partial y}\right) - \lambda_e\frac{e}{k}\frac{A_7}{A_5}\left(E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x}\right) + \lambda_e\frac{e}{k}\frac{A_8}{A_5}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) \\
q_y^{\,e} &= -\lambda_e\left(\frac{A_6}{A_5}\frac{\partial T_e}{\partial x} + \frac{\partial T_e}{\partial y}\right) - \lambda_e\frac{e}{k}\frac{A_7}{A_5}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) - \lambda_e\frac{e}{k}\frac{A_8}{A_5}\left(E_x + vB + \frac{1}{en}\frac{\partial p_e}{\partial x}\right)
\end{aligned} \tag{9}$$

The transport coefficients are

$$\begin{aligned}
\sigma &= \frac{ne^2\tau_e}{m_e}A_1 \qquad \eta_i = \frac{{}^5/_6 kT_i\tau_i n}{1 + {}^{25}/_9 H_i^2}, \qquad \lambda_i = \frac{{}^{25}/_8 n\tau_i}{1 + {}^{25}/_{16}H_i^2}\frac{k^2}{m}T_i \\
\eta_e &= \frac{5}{3(2+\sqrt{2})}\,\frac{kT_e\tau_e n}{1 + 100/9 + (2+\sqrt{2})^2 H_e^2}, \qquad \lambda_e = \frac{5}{2}\frac{nk^2}{m_e}T_e\tau_e A_5
\end{aligned} \tag{10}$$

In these expressions

$$\begin{aligned}
&A_1 = \nabla(1.93 + 1.07H_e^2), \qquad A_2 = \nabla(4.69H_e + 1.07H_e^3) \\
&A_3 = \nabla(1.55 - 1.6H_e^2), \qquad A_4 = \Delta 4.6H_e \\
&A_5 = \nabla(2.58 + 0.391H_e^2), \qquad A_6 = \nabla(6.63H_e + 1.06H_e^3) \\
&A_7 = \nabla(2.55 + 0.42H_e^2), \qquad A_8 = \nabla(6.53H_e + 1.07H_e^3) \\
&\nabla = (1 + 6.73H_e^2 + 1.67H_e^4)^{-1}, \qquad H_e = \omega_e\tau_e, \qquad H_i = \omega_i\tau_i
\end{aligned} \tag{11}$$

Here H_e and H_i are the Hall parameters, ω_e and ω_i are the Larmor frequencies, and τ_i and τ_e are the mean times between collisions of ions with one another and collisions of electrons with ions.

The system (1)-(5) of gasdynamic equations is supplemented by the following relations:

electrodynamic

$$\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} = 0, \quad \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} = 0 \tag{12}$$

equations of state

$$p_i = nkT_i, \quad p_e = nkT_e \tag{13}$$

the relation

$$p = p_i + p_e \tag{14}$$

The equations can be reduced to dimensionless form in order to compare individual terms. We select the scales: longitudinal dimensions x_0, transverse dimensions y_0, longitudinal and transverse velocity components u_0 and v_0, respectively, concentration n_0, temperatures T_0, pressures $p_0 = n_0 k T_0$, electric field intensity components E_0, magnetic field B_0, times between collisions τ_{e0} and τ_{i0}. The scales for the Hall parameters are

$$H_{e0} = \frac{eB_0}{m_e}\tau_{e0}, \quad H_{i0} = \frac{eB_0}{m}\tau_{i0}$$

The scales for the functions A_k are determined by (11) in terms of the scale H_{e0}. The scales for the gas kinetic quantities σ_0, η_{i0}, η_{e0}, λ_{i0}, and λ_{e0} are connected by (10) with the scales n_0, T_0, τ_{e0}, τ_{i0}, H_{e0}, H_{i0}, and A_{10}.

In transforming the equations to dimensionless form we introduce the following similarity criteria:

$$R_i = \frac{u_0 m n_0 x_0}{\eta_{i0}}, \quad P_i = \frac{\eta_{i0} c_p}{\lambda_{i0}} \quad \left(c_p = \frac{5}{2}\frac{k}{m}\right),$$

$$R_e = \frac{u_0 m n_0 x_0}{\eta_{e0}}, \quad P_e = \frac{\eta_{e0} c_p}{\lambda_{e0}}, \quad \Phi = \frac{eE_0 x_0}{kT_0}, \quad K = \frac{E_0}{Bu_0}$$

$$M = \frac{u_0}{\sqrt{\varkappa T_0 k / m}}, \quad M^D = \frac{\sigma_0 E_0}{en_0 \sqrt{\varkappa T_0 k / m}} \quad \left(\varkappa = \frac{5}{3}\right)$$

Here R_e, R_i are the electron and ion Reynolds numbers, respectively; P_e, P_i are the electron and ion Prandtl numbers; Φ is the electric field criterion; K is the loading criterion; M is the Mach number; M^D is the diffusion Mach number, in which the current scale is $\sigma_0 E_0$. For y_0 and v_0 we take the relations

$$y_0 = \frac{x_0}{\sqrt{R_i}}, \quad v_0 = \frac{u_0}{\sqrt{R_i}}$$

Then (1) in dimensionless form with account for (7) and (8) has the form (the overscore denotes quantities referred to their scales)

$$\begin{aligned}
\bar{n}\left(\bar{u}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{u}}{\partial \bar{y}}\right) = &-\frac{\partial \bar{p}}{\partial \bar{x}}\frac{1}{\varkappa M^2} + \bar{B}\bar{n}\bar{\tau}_e\left[\bar{A}_1\left(\bar{E}_y - \bar{u}\bar{B}\frac{1}{K} + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\right.\\
&\left. + \bar{A}_2\frac{A_{20}}{A_{10}}\left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}} + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{x}}\frac{1}{\Phi}\right)\right]\frac{M^D\Phi}{\varkappa M^3 K}\\
&+ \bar{B}\bar{n}\bar{\tau}_e\left(\frac{A_{30}}{A_{10}}\bar{A}_3\frac{\partial \bar{T}_e}{\partial \bar{y}}\sqrt{R_i} + \frac{A_{40}}{A_{10}}\bar{A}_4\frac{\partial \bar{T}_e}{\partial \bar{x}}\right)\frac{M^D}{\varkappa M^3 K}\\
&+ \frac{\partial}{\partial \bar{x}}\left\{\bar{\eta}_i\left[\frac{4}{3}\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{2}{3}\frac{\partial \bar{v}}{\partial \bar{y}} + \bar{H}_i H_{i0}\frac{5}{3}\left(\frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}} + \frac{\partial \bar{u}}{\partial \bar{y}}\sqrt{R_i}\right)\right]\right\}\frac{1}{R_i}\\
&+ \frac{\partial}{\partial \bar{y}}\left\{\bar{\eta}_i\left[\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{R_i} - \bar{H}_i H_{i0}\frac{5}{3}\left(\frac{\partial \bar{u}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}} - \frac{\partial \bar{v}}{\partial \bar{y}}\frac{1}{R_i}\right)\right]\right\}
\end{aligned} \tag{15}$$

Let us examine the case in which the Reynolds criterion for the ions is much larger than the other dimensionless criteria and their combinations M, M^D, Φ, K, H_{i0}, H_{e0}, A_{20}/A_{10}, and so on, and we assume that it has been possible to select the scales so that all the dimensionless quantities and their gradients are of order unity. Then in (15) we can neglect terms containing the factors $1/\sqrt{R_i}$ and $1/R_i$ in comparison with the terms containing unity as a multiplier, and also terms containing unity as a multiplier in comparison with similar terms containing the factor $\sqrt{R_i}$. Dropping the corresponding terms and returning to dimensional variables, we obtain

$$mn\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\eta_i\frac{\partial u}{\partial y}\right) + j_y B \tag{16}$$

where the transverse current density is

$$j_y = \sigma\left(E_y - uB + \frac{1}{en_e}\frac{\partial p_e}{\partial y} + \frac{A_2}{A_1}E_x + \frac{k}{e}\frac{A_3}{A_1}\frac{\partial T_e}{\partial y}\right) \tag{17}$$

Proceeding similarly with (2), with account for (6) and (8) we obtain

$$\frac{\partial \bar{p}}{\partial \bar{y}} = \bar{B}\bar{n}\bar{\tau}_e\left(\bar{A}_2\frac{A_{20}}{A_{10}}\frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{y}} + \bar{A}_4\frac{A_{40}}{A_{10}}\frac{\partial \bar{T}_e}{\partial \bar{y}}\right)\frac{M^D}{\varkappa M^3 K} \tag{18}$$

Consequently, the pressure can be assumed constant across the boundary layer if the criteria combination is sufficiently small. Here S is the magnetic interaction parameter.

$$\frac{M^D}{\varkappa M^3 K} = \frac{S}{\Phi} \qquad \left(S = \frac{\sigma_0 E_0 B_0 x_0}{m n_0 u_0^2}\right) \tag{19}$$

After suitable simplifications the ion energy equation takes the form

$$mnc_p\left(u\frac{\partial T_i}{\partial x} + v\frac{\partial T_i}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda_i \frac{\partial T_i}{\partial y}\right) + u\frac{\partial p_i}{\partial x} + v\frac{\partial p_i}{\partial y} + \eta_i\left(\frac{\partial u}{\partial y}\right)^2 + \frac{3km_e}{m}\frac{n}{\tau_e}(T_e - T_i) \tag{20}$$

In view of the importance of the electron energy equation, we write it out in complete dimensionless form

$$\begin{aligned}
\frac{3}{2}\bar{n}\left(\bar{u}\frac{\partial \bar{T}_e}{\partial \bar{x}} + \bar{v}\frac{\partial \bar{T}_e}{\partial \bar{y}}\right) = {} & \frac{\partial}{\partial \bar{x}}\left\{\bar{\lambda}_e\left(\frac{\partial \bar{T}_e}{\partial \bar{x}} - \sqrt{R_i}\,\frac{A_{60}}{A_{50}}\frac{\bar{A}_6}{\bar{A}_5}\frac{\partial \bar{T}_e}{\partial \bar{y}}\right)\frac{\varkappa}{(\varkappa-1)R_e}\frac{1}{P_e}\right. \\
& + \frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_7}{\bar{A}_1}\left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}} + \frac{1}{\bar{n}}\frac{\partial p_e}{\partial \bar{x}}\frac{1}{\Phi}\right)\frac{M^D}{M}\frac{A_{70}}{A_{10}} \\
& \left. - \frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_8}{\bar{A}_1}\left(\bar{E}_y - \bar{u}\bar{B}\frac{1}{K} + \frac{1}{\bar{n}}\frac{\partial p_e}{\partial \bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{M^D}{M}\frac{A_{80}}{A_{10}}\right\} \\
& + \frac{\partial}{\partial \bar{y}}\left\{\bar{\lambda}_e\left(\frac{\bar{A}_6}{\bar{A}_5}\frac{A_{60}}{A_{50}}\frac{\partial \bar{T}_e}{\partial \bar{x}}\frac{1}{\sqrt{R_i}} + \frac{\partial \bar{T}_e}{\partial \bar{y}}\right)\frac{\varkappa}{\varkappa-1}\frac{R_i}{P_e R_e} + \frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_7}{\bar{A}_1}\left(\bar{E}_y - \bar{u}\bar{B}\frac{1}{K}\right.\right. \\
& \left. + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{M^D}{M}\frac{A_{70}}{A_{10}}\sqrt{R_i} + \frac{5}{2}\bar{\sigma}\bar{T}_e\frac{\bar{A}_8}{\bar{A}_1}\left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}\right. \\
& \left.\left. + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{x}}\frac{1}{\Phi}\right)\frac{M^D}{M}\frac{A_{80}}{A_{10}}\sqrt{R_i}\right\} - \bar{p}_e\left(\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}}\right) + \frac{\partial \bar{u}}{\partial \bar{x}}\left\{\bar{\eta}_e\left[\frac{4}{3}\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{2}{3}\frac{\partial \bar{v}}{\partial \bar{y}}\right.\right. \\
& \left.\left. - \bar{H}_e\frac{10H_{e_0}}{3(2+\sqrt{2})}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\sqrt{R_i} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}}\right)\right]\right\}\frac{\varkappa M^2}{R_e} + \left(\frac{\partial \bar{u}}{\partial \bar{y}}\sqrt{R_i} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}}\right) \\
& \times\left\{\bar{\eta}_e\left[\frac{\partial \bar{u}}{\partial \bar{y}}\sqrt{R_i} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}} + \bar{H}_e\frac{10H_{e0}}{3(2+\sqrt{2})}\left(\frac{\partial \bar{u}}{\partial \bar{x}} - \frac{\partial \bar{v}}{\partial \bar{y}}\right)\right]\right\}\frac{\varkappa M^2}{R_e} \\
& + \frac{\partial \bar{v}}{\partial \bar{y}}\left\{\eta_e\left[\frac{4}{3}\frac{\partial \bar{v}}{\partial \bar{y}} - \frac{2}{3}\frac{\partial \bar{u}}{\partial \bar{x}} + \bar{H}_e\frac{10H_{e0}}{3(2+\sqrt{2})}\left(\frac{\partial \bar{u}}{\partial \bar{y}}\sqrt{R_i} + \frac{\partial \bar{v}}{\partial \bar{x}}\frac{1}{\sqrt{R_i}}\right)\right]\right\}\frac{\varkappa M^2}{R_e} \\
& + \left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}}\right)\bar{\sigma}\left\{\left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}} + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{x}}\frac{1}{\Phi}\right)\frac{\Phi M^D}{M}\right. \\
& - \frac{\bar{A}_2}{\bar{A}_1}\left(\bar{E}_y - \bar{u}\bar{B}\frac{1}{K} + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{y}}\frac{\sqrt{R_i}}{\Phi}\right)\frac{\Phi M^D}{M}\frac{A_{20}}{A_{10}} + \left(\frac{\bar{A}_3}{\bar{A}_1}\frac{\partial \bar{T}_e}{\partial \bar{x}}\frac{A_{30}}{A_{10}}\right. \\
& \left.\left. - \frac{\bar{A}_4}{\bar{A}_1}\frac{\partial \bar{T}_e}{\partial \bar{y}}\frac{A_{40}}{A_{10}}\sqrt{R_i}\right)\frac{M^D}{M}\right\} + (\bar{E}_y - \bar{u}\bar{B})\bar{\sigma}\left\{(\bar{E}_y - \bar{u}\bar{B}\right. \\
& + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{y}}\frac{\sqrt{R_i}}{\Phi}\Big)\frac{\Phi M^D}{M} + \frac{\bar{A}_2}{\bar{A}_1}\left(\bar{E}_x + \bar{v}\bar{B}\frac{1}{K\sqrt{R_i}} + \frac{1}{\bar{n}}\frac{\partial \bar{p}_e}{\partial \bar{x}}\frac{1}{\Phi}\right)\frac{\Phi M_D}{M}\frac{A_{20}}{A_{10}} \\
& \left. + \left(\frac{\bar{A}_3}{\bar{A}_1}\frac{\partial \bar{T}_e}{\partial \bar{y}}\frac{A_{30}}{A_{10}}\sqrt{R_i} + \frac{\bar{A}_4}{\bar{A}_1}\frac{\partial \bar{T}_e}{\partial \bar{x}}\frac{\bar{A}_{40}}{\bar{A}_{10}}\right)\frac{M^D}{M}\right\} - 3\frac{\bar{n}}{\bar{\tau}_e}(\bar{T}_e - \bar{T}_i)\frac{\Phi A_{10}}{\varkappa M M^D}
\end{aligned} \tag{21}$$

We assume that the criterion R_e is sufficiently large that

$$R_e P_e \gtrsim R_i, \quad R_e > H_{e0}\sqrt{R_i}, \quad H_{e0} < \sqrt{R_i} \tag{22}$$

In this case the Hall numbers for the electrons can therefore be large but not infinite.

Neglecting terms containing a factor of unity or $\sqrt{R_i}$ in comparison with analogous terms containing the factors $\sqrt{R_i}$ and R_i, respectively, with account for inequalities (22), returning to dimensional variables and introducing the heat capacity c_p, we obtain

$$mnc_p\left(u\frac{\partial T_e}{\partial x} + v\frac{\partial T_e}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda_e\frac{\partial T_e}{\partial y} + \frac{5}{2}\frac{j_y^q}{e}kT_e\right) + u\frac{\partial p_e}{\partial x} + v\frac{\partial p_e}{\partial y} + j_x E_x + j_y E_y + \eta_e\left(\frac{\partial u}{\partial y}\right)^2 - 3k\frac{m_e}{m}\frac{n}{\tau_e}(T_e - T_i) \tag{23}$$

Here

$$j_x = \sigma\left\{E_x - \frac{A_2}{A_1}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) - \frac{k}{e}\frac{A_4}{A_1}\frac{\partial T_e}{\partial y}\right\} \tag{24}$$

is the axial (Hall) current density, and

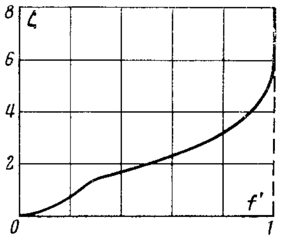

Fig. 1

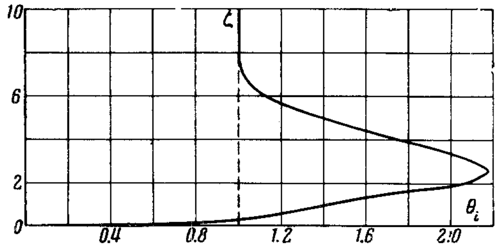

Fig. 2

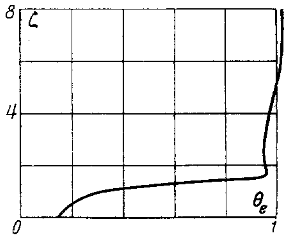

Fig. 3

$$j_y^q = \sigma\left\{\frac{A_7}{A_1}\left(E_y - uB + \frac{1}{en}\frac{\partial p_e}{\partial y}\right) + \frac{A_8}{A_1}E_x\right\} \tag{25}$$

is the density of the current transported by enthalpy in the transverse direction.

The dimensionless form of the aerodynamic equations

$$\frac{\partial \bar{E}_y}{\partial \bar{x}} - \frac{\partial \bar{E}_x}{\partial \bar{y}}\sqrt{R_i} = 0, \quad \frac{\partial \bar{j}_x}{\partial \bar{x}} + \frac{\partial \bar{j}_y}{\partial \bar{y}}\sqrt{R_i} = 0$$

makes it possible to conclude that E_x and j_y are constants across the boundary layer.

The problem of the plasma boundary layer on the electrodes will be completely defined if we specify the following boundary conditions:

for $y = 0$

$$u = v = 0, \quad T_i = T_{iw}(x), \quad T_e = T_{ew}(x) \tag{26}$$

for $\bar{y} = \infty$

$$u = U_s(x), \quad T_i = T_{is}(x), \quad T_e = T_{es}(x), \quad j_y = j_{ys}(x), \quad E_x = E_{xs}(x) \tag{27}$$

One of the possible methods for integrating the system of boundary layer equations for the two-temperature plasma is the method of finding the locally self-similar flows, in which the dependence of all the quantities on the longitudinal coordinate is accounted for through variation of the outer flow parameters. We introduce the independent variables

$$\xi = \int_0^{\bar{x}} \bar{U}_s d\bar{x}, \quad \zeta = \frac{\bar{U}_s\sqrt{R_i}}{\sqrt{2\xi}}\int_0^{\bar{y}} \bar{n}d\bar{y}$$

$$\left(\bar{U}_s = \frac{U_s}{U_{s0}},\ \bar{n} = \frac{n}{n_s},\ \bar{x} = \frac{x}{L},\ \bar{y} = \frac{y}{L},\ R_i = \frac{U_{s0}Lmn_{s0}}{\eta_{is0}}\right)$$

(subscript 0 applies to the initial section x = 0) and the functions

$$f' = \frac{u}{U_s},\ \theta_e = \frac{T_e}{T_{es}},\ \theta_i = \frac{T_i}{T_{is}}$$

We examine only an isothermal accelerating outer flow with the constant parameters

$$T_{i_s} = \text{const},\ T_{es} = \text{const},\ p = \text{const},\ E_{x_s} = \text{const},\ j_{y_s} = \text{const},\ j_{x_s} = 0$$

Then (16), (20), and (23) reduce to the form

$$ff'' + \frac{2\xi S\bar{T}_{es}}{\bar{U}_s^3}\theta_e + \frac{2\xi S\bar{T}_{is}}{\bar{U}_s^3}\theta_i - \frac{2\xi}{\bar{U}_s}\frac{d\bar{U}_s}{d\xi}f'^2 + (\bar{\eta}_{i1}f'')' = 0 \tag{28}$$

$$f\theta_i' + (\varkappa - 1)M^2\frac{\bar{U}_s^2}{T_{is}}\bar{\eta}_{i1}f''^2 + (\bar{\lambda}_{i1}\theta_i')' + \frac{\varkappa - 1}{\varkappa}\bar{T}_{es}f\frac{\theta_i\theta_e' - \theta_e'\theta_i'}{\theta_e\bar{T}_{es} + \theta_i\bar{T}_{is}}$$

$$+ 6\frac{\varkappa - 1}{\varkappa^3}\frac{\Phi^2\xi}{M^4SK}\frac{A_{1s0}^2 + A_{2s0}^2}{\bar{U}_s^2A_{2s0}\bar{T}_{is}}\frac{\theta_e\bar{T}_{es} - \theta_i\bar{T}_{is}}{\theta_e^{3/2}(\theta_e\bar{T}_{es} + \theta_i\bar{T}_{is})} = 0 \tag{29}$$

$$f\theta_e' + (\varkappa - 1)M^2\frac{\bar{U}_s^2}{\bar{T}_{es}}\bar{\eta}_{e1}f''^2 + (\bar{\lambda}_{e1}\theta_e')' + \frac{\varkappa - 1}{\varkappa}\bar{T}_{is}f\frac{\theta_e\theta_i' - \theta_i\theta_e'}{\theta e\bar{T}_{es} + \theta_i\bar{T}_{is}}$$

$$+ 2(\varkappa - 1)\frac{M^2SK}{\bar{T}_{es}}\frac{\xi}{\bar{U}_s^2}(\theta_e\bar{T}_{es} + \theta_i\bar{T}_{is})\left[\bar{\sigma}\left(1 + \frac{A_2^2}{A_1^2}\right) - 2\frac{A_2}{A_1} + \frac{1}{\bar{\sigma}}\right]$$

$$+ (\varkappa - 1)\sqrt{2\xi R_i}\,\frac{M^2SK}{\Phi\bar{U}_s}\left\{\theta_e'\left[-\frac{\theta_i\bar{T}_{is}}{\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is}} + \bar{\sigma}\left(\frac{A_2A_3}{A_1^2} - \frac{A_4}{A_1}\right)\right.\right.$$

$$\left.\left. + \frac{5}{2}\frac{j_y^q}{j_y} - \frac{A_3}{A_1}\right] + \theta_i'\frac{\theta_e\bar{T}_{is}}{\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is}}\right\} - 6\,\frac{(\varkappa-1)\Phi^2\xi}{\varkappa^3M^4SK}\,\frac{A_{1s0}^{\ 2}+A_{2s0}^{\ 2}}{\bar{U}_s^{\ 2}A_{2s0}\bar{T}_{es}}$$

$$\times\frac{\theta_e\bar{T}_{es}-\theta_i\bar{T}_{is}}{\theta_e^{3/2}(\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is})} = 0 \qquad (30)$$

The plasma transport properties

$$\bar{\eta}_{i1} = \frac{\eta_i/\eta_{is}}{\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is}}, \quad \bar{\lambda}_{i1} = \frac{\lambda_i/\lambda_{is}}{(\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is})P_i}, \quad \bar{\eta}_{e1} = \frac{\eta_e/\eta_{es}}{\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is}}\,\frac{R_i}{R_e},$$

$$\bar{\lambda}_{e1} = \frac{\lambda_e/\lambda_{es}}{(\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is})}\,\frac{R_i}{P_eR_e}, \qquad \bar{\sigma} = \frac{\sigma}{\sigma_{00}} = \frac{\sigma E_x}{j_y} = \frac{\sigma}{\sigma_s}\,\frac{A_{2s}/A_{1s}}{1+A_{2s}^2/A_{1s}^2}$$

in accordance with (10) and with account for the definitions [4]

$$\tau_i = (4\pi\varepsilon)^2\,\frac{3\sqrt{m}\,(kT_i)^{3/2}}{4\sqrt{\pi}\lambda_ie^4n}, \qquad \tau_e = (4\pi\varepsilon)^2\,\frac{3\sqrt{m_e}\,(kT_e)^{3/2}}{4\sqrt{2\pi}\lambda_ee^4n}$$

are expressed through θ_e, θ_i, H_e, and H_i, where

$$H_i = \theta_i^{3/2}(\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is})\,H_{is}, \qquad H_e = \theta_e^{3/2}(\theta_e\bar{T}_{es}+\theta_i\bar{T}_{is})\,H_{es}$$

Assuming j_y^q to be constant, we can find

$$\frac{j_y^q}{j_y} = \frac{A_{1s}A_{7s}+A_{2s}A_{8s}}{A_{1s}^2+A_{2s}^2}$$

The problem's similarity criteria will be the quantities

$$S = \frac{j_yBL}{mn_sU_{s0}^2}, \qquad M = \frac{U_{s0}}{\sqrt{\varkappa kT_s/m}}, \qquad \Phi = \frac{eE_xL}{kT_s}$$

$$K = \frac{E_x}{U_{s0}B}, \qquad \bar{T}_{es} = \frac{T_{es}}{T_s}, \qquad \bar{T}_{is} = \frac{T_{is}}{T_s}$$

$$R_i = \frac{U_{s0}mn_sL'}{\eta_{is}}, \qquad R_e = \frac{U_{s0}mn_sL}{\eta_{es}}, \qquad P_e = \frac{\eta_{es}c_p}{\lambda_{es}}$$

$$P_i = \frac{\eta_{is}c_p}{\lambda_{is}}, \qquad H_{is} = \omega_i\tau_{is}, \qquad H_{es} = \omega_e\tau_{es}$$

We note that the magnetic interaction parameter S is connected with the diffusion (current) Mach number by the relation

$$S = \frac{M^j\Phi}{\varkappa M^3K_i}, \qquad M_j = \frac{j_y}{en_s\sqrt{\varkappa kT_s/m}} = \frac{\sigma_{00}E_x}{en_s\sqrt{\varkappa kT_s/m}} = \frac{\sigma_{00}}{\sigma_s}M^D$$

The problem's boundary conditions have the form

$$\begin{aligned} &f(0) = f'(0) = 0, \quad f'(\infty) = 1 \\ &\theta_e(0) = \theta_{ew} = \text{const}, \quad \theta_e(\infty) = 1 \\ &\theta_i(0) = \theta_{iw} = \text{const}, \quad \theta_i(\infty) = 1 \end{aligned} \qquad (31)$$

As an example Figs. 1-3 show the distributions u/U_s, Θ_i, and Θ_e in the boundary layer of a two-temperature fully ionized argon plasma for $\xi = 0$, obtained as a result of numerical calculation on an M-20 computer with the following values of the defining parameters

$$\begin{gathered} M = 2.45, \quad K = 1.036, \quad \bar{T}_{es} = 0.833, \quad \bar{T}_{is} = 0.167 \\ R_i = 1.565\cdot10^4, \quad R_e = 3.61\cdot10^{10}, \quad P_i = 0.619, \quad P_e = 1.925\cdot10^{-5} \\ H_{is} = 0.265, \quad H_{es} = 438, \quad \Theta_{ew} = 0.15, \quad \Theta_{iw} = 0.15 \end{gathered}$$

Certain of the peculiarities observed in the resulting profiles are apparently associated with the behavior of the transport properties in the strong magnetic field.

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